Natural Convention in a Horizontal Cylindrical Annulus with Equally Spaced Radially Divergent Longitudinal Solid Fins: Part I

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Abstract

A numerical investigation was carried out to study laminar natural convection heat transfer in a stationary horizontal concentric cylindrical annulus with six equally spaced radially divergent round tip longitudinal solid metallic fins attached on the external surface of the inner cylinder. The surface of the inner cylinder as well as the surface of the outer cylinder was assumed to be at a uniform temperature. The non-dimensional thermal conductivity of the metallic fin to fluid was assumed to be high. Water was selected as the working fluid. The Rayleigh number based on the width of the annulus gap was varied from 10^3 to 10^6 and the ratio of the fin height to the annulus gap was varied as 0, 0.2, 0.4 and 0.6. The velocity vectors and the isotherm plots were depicted to investigate the fluid flow behavior and the heat transfer inside the annulus. It was found that the heat transfer rate from the inner cylinder increased with the increased of both the Rayleigh number and the fin height. The variations of the local equivalent thermal conductivity (ratio of Nusselt numbers due to convection and conduction heat transfer from the inner cylinder) were also presented. Four empirical correlations were developed for the average equivalent thermal conductivity as a function of Rayleigh number for three different fin heights and for the annulus without a fin. The heat transfer rate was much more pronounced in all of the fins-fitted geometries compared to the case with no fins at the same Rayleigh number.

Keywords: *longitudinal metallic solid divergent fins, laminar natural convection, horizontal annulus, and average equivalent thermal conductivity.*

NOMENCLATURE

g	Gravitational acceleration, $m.s^{-2}$
h	Convective heat transfer coefficient, $W.m^{-2}.^0C^{-1}$
H	Fin height, m
K_f	Fluid thermal conductivity, $W.m^{-1}.^0C^{-1}$
K_s	Solid thermal conductivity, $W.m^{-1}.^0C^{-1}$
K_{eq}	Local equivalent thermal conductivity = $\frac{Nu_L}{Nu_0}$
$\overline{K}_{eq}$	Circumferential average equivalent thermal conductivity = $\frac{\overline{N}_u}{Nu_0}$
L	Gap width of the annulus = $(r_o - r_i)$, m
Nu_0	Nusselt number for conduction between the annuli
Nu_L	Local Nusselt number
$\overline{Nu}_{avg}$	Circumferential average Nusselt number based on cylinder radius
P	Pressure, Pa
Pr	Prandtl number $=\frac{V_f}{\alpha_f}$
r_i	Radius of inner cylinder, m
r_o	Radius of outer cylinder, m
R_i	Dimensionless radius of inner cylinder
R_o	Dimensionless radius of outer cylinder
Ra	Rayleigh number = $\frac{g\beta_f(T_i-T_0)r_i^3\rho}{\mu\alpha_f}$
T	Temperature, K
T_0	Temperature on cold wall, K

T_i	Temperature on hot wall, K
T_{ref}	Reference temperature (T_0), K
φ	Dimensionless temperature $=\frac{T-T_{ref}}{T_i-T_0}$
φ_0	Dimensionless temperature on outer cylinder wall.
φ_i	Dimensionless temperature on inner cylinder wall.
Φ	Transported scalar
u, v	Interstitial velocity components along θ and r directions respectively, ms^{-1}
U, V	Dimensionless interstitial velocity components along θ and r directions, respectively
θ , r	Polar coordinates, degree and m

Greek symbols

α_f	Thermal diffusivity, m^2s^{-1}
β_f	Coefficient of thermal expansion, $\frac{1}{T}$, K^{-1}
v_f	Kinematic viscosity, m^2s^{-1}
ρ	Density, $kg.m^{-3}$
μ	Dynamic viscosity, $kg.m^{-1}.s^{-1}$

Subscripts

i	Inner cylinder
o	Outer cylinder
ref	Reference value
F	Fluid
S	Solid

1.0 INTRODUCTION

Over the years, the natural convection heat transfer in enclosures has been a subject of extensive research. Among them, convective heat transfer in horizontal annuli has attracted many attentions in engineering systems due to its wide range of applications such as in concentrating solar collectors, aircraft cabin insulation, thermal energy storage devices, cooling of underground electric transmission cables, air conditioning, refrigeration, gas turbines, compressors, and in some aspect of the design of nuclear reactors. So far a large number of research works has been published on the investigation of natural convection in horizontal annuli without fins [1-4]. Previous investigations in finless annuli showed that the heat transfer in a horizontal annulus is limited by the outside surface of the inner cylinder.

Thereafter, the internal fins are used as an alternative to increase the thermal performance of such systems. Heat transfer from the finned surfaces can be enhanced through the accomplishments of the following practical techniques: (1) increasing the surface area to volume ratio, (2) increasing the thermal conductivity of the fin, and (3) increasing the convective heat transfer coefficient between the surface of the solid fin and the surrounding fluid. It is obvious that the presence of internal fin (extended surface) alters the fluid flow pattern since the fin acts as an obstacle or a partition. This modification of the flow pattern and the extension of the heat exchange area between the fluid and heated solid surfaces significantly influence the total heat transfer rate. At the same time, the volume of the fin cannot be increased beyond a certain limit which may lead to the fluid flow constraint within the enclosure. In view of the above-mentioned constraint, the design of the fin i.e., the surface of the fin in contact with the fluid is an important factor, Despite the shape and height of the fin, in a finned cavity the placement of the fin with respect to the gravity field has also a dominant effect on the heat transfer mechanism so far the natural convection is concerned. In some angular positions of the fin with respect to the flow direction, the buoyancy forces may not be strong enough to trigger significant convection. For example, when a fin is positioned horizontally with respect to the gravity field inside an annulus, the top surface of the fin results in a Bernard-type of heat convection. As a result, this orientation of fin significantly enhances the heat transfer rate from the base to the tip of the fin. This happens due to the unstable density gradient therein. At the bottom surface of the horizontally positioned fin, a thermal stratification occurs which leads to a stable density gradient there. As a result, the low conductive mode of heat transfer prevails underneath the fin surface. When the fin is placed in an inclined position with respect to the horizontal plane or positioned vertically, the heat transfer phenomenon is completely different from the above-explained orientation. For a fixed geometry and for a particular working fluid, the net enhancement of the heat transfer rate depends on the combinations of the number of fins used, the fin height, the spacing between the fins, and the Rayleigh number.

An explanation is needed with regard to the thermal conductivity of the metallic fins with respect to the fluid conductivity. It has been noticed that many previous studies [5] which have considered fins attached to the various surfaces of the enclosure have assumed that the conductivity of the fins is practically infinity with respect to the conductivity of the surrounding fluid. With the above assumption, the authors have unrealistically considered the fin surfaces to be at the same temperature of the surfaces where the fins have been attached. One should realize that the heat transfer between the fin and the working fluid is a conjugate heat transfer problem and a priori imposition of the temperature of the fin may lead to an unrealistic high heat transfer rate.

Compared to the bare annulus, very limited work on natural convection heat transfer in horizontal finned annuli has been reported in the open literature. The increase in the complexity that arises due to the strong interaction between the thermal boundary layer on the fin surfaces and the adjacent fluid makes it difficult to obtain a converged numerical solution of the problem of natural convection in a geometrically complicated finned annulus. It is observed that over the past fifteen years or so, most of the studies related to natural convection in finned-annulus seem to have been numerically investigated using the commercially available CFD codes like ANSYS Fluent, COMSOL, OpenFOAM, etc. Depending on the users' knowledge of CFD and the applied code, it has been seen that in many cases contradictory results have been published in the literature. The present authors have used in their numerical investigation a well verified in-house CFD code which was written from scratch by the authors. The present authors are therefore confident about the accuracy of their numerically predicted results of the studied problem. The following is a brief review of natural convection in a solid-finned horizontal annulus:

Among the earlier studies, the work of Chai and Patankar [5] is a pioneering work in the field of natural convection in a horizontal annulus with solid fins. The above authors numerically studied the flow and heat transfer for an annulus with equally spaced six radial fins of constant thickness. They studied the effects of two fin orientations. In one configuration, the two fins out of the six fins were placed in the upper and lower vertical symmetry planes and the rest four were placed at an equal angular distance from the bottom or top fin. In the second case the two fins were positioned at the right and left horizontal symmetry planes and the rest four fins were placed at an equal angular position from the horizontal fin either left or right. The two ratios of fin height to the annulus gap were selected. One had a value of 0.4 and the other one was assigned a value of 0.6. The constant temperature boundary condition was considered for the fins which were equal to the temperature of the inner cylinder. This assumption technically implied that the conductivity of the fins is infinite, which is not a sound assumption so far the natural convection heat transfer is concerned. It was observed that the orientation of the internal fins did not have an important effect on the average Nusselt number values, but the blockage due to the fins had a significant effect on the flow and temperature fields and therefore on the rate of heat transfer. Their results indicate that the average Nusselt number increases with the increasing Rayleigh number and decreases with the increase of the fin height. The authors did not show how they calculated the local and average Nusselt numbers on the inner cylinder in the presence of six solid fins. Because of this lack of the above information, it is difficult to be confident in their predicted Nusselt number results.

Silva and Gosselin [6] studied numerically by employing finite element method the steady-state laminar natural convection in a three-dimensional differentially heated cubic enclosure with a high conductivity rectangular fin attached to the hot wall of the cavity. The computations were carried out for a wide range of Rayleigh number based on the enclosure height (Ra) and for the two different values of the fin volume. They investigated the effect of the aspect ratio and horizontal length of the fin. The results indicated that for a large volume fraction of fin, the fin aspect ratio does not play an important role. The horizontal length of the fin is found to have a significant effect on heat transfer to the fluid and the heat flux transferred to the fluid is shown to increase monotonically with respect to the fin length. When the volume fraction of the fin is relatively small, the authors noted that the average heat flux increases with the aspect ratio of the fin, as well as the horizontal length of the fin.

Alshahrani and Zeitoun [7] investigated numerically the steady laminar natural convection heat transfer between the two horizontal concentric cylinders attached with two solid fins on the outer surface of the inner cylinder. They investigated the effects of various inclination angles of the fins, and the Rayleigh number up to the value of 5×10^4. So far the heat transfer is concerned they found that the increase in fin height contributed more to the conduction-dominated zone than to the convection-dominated zone. The above authors have reported that when the fin height to annulus gap is 75 per cent, in the conduction-dominated zone the heat transfer is increased by 100 per cent while in the convection-dominated zone it increased by 20 per cent compared to the bare annulus. For the change in the fin inclination angle from the horizontal position to the vertical position, a 25 per cent increase in heat transfer is reported. It appears that the authors have got erroneous heat transfer results for the bottom part of the annulus compared to the upper part.

Al-Kouz et al. [8] carried out a numerical simulation for two-dimensional steady natural convection heat transfer inside a concentric horizontal annulus in which the inner cylinder was attached with two solid fins for the low-pressure laminar flows. They studied the effect of rarefaction of the gas on the flow and heat transfer characteristics of such flows. The effect of Knudsen number (the ratio of the mean free path to the characteristic length of the geometry of interest), the Ralyeigh number, the fin inclination angle and the conductivity ratio on the flow and heat transfer were also investigated. It is found that the Nusselt number depends inversely on the Knudsen number and directly on the Rayleigh number. It is further observed that an attachment of a solid fin to the inner cylinder surface enhances the heat transfer rate for such low-pressure flows. Upon increasing the tilt angle of the fin a better heat transfer is seen to occur. The authors concluded that by increasing the conductivity ratio of the fin to fluid, a better heat transfer is achieved.

Recently, a comprehensive review paper was published by Khadanga1 and Rao [9] which investigated the effect of various parameters such as the fin geometry, fin material, surface roughness and slot sizes on the heat transfer rate of the fin. By reviewing the rectangular, circular and curved shaped fins made up of aluminum alloy AA-204, aluminum alloy AA-6061 and magnesium alloy of two different thicknesses (3 mm and 2.5 mm), it was noticed that a considerable change in heat transfer rate occurred while the parameters were varying. The review also showed that the surface roughness and the slots or holes or gaps in fins have a great influence on the heat transfer rate. The heat transfer rate increased with the increase in the surface roughness and the slot size. It was further found that with the increase in the slot size the heat transfer rate increased up to some extent and then it decreased. No explanation was provided for the latter anomalies.

Although not reviewed in this work, a number of studies do exist in the literature with regard to the natural convection with solid fins in an annulus where various types of nano-fluids have been used. Since the employment of a high conductivity nano-fluid is an alternative technique for enhancing the heat transfer rate, but there is no relevance to those works with the present studied problem. Hence, those studies are not reviewed here.

From the above literature review, one can see that a number of reports are available regarding natural convection heat transfer for various shapes of fins, such as rectangular, triangular, annular, concave, parabolic, etc with regard to various shaped-enclosures. Until now, not a single work seems to exist for the radially divergent longitudinal fins in an enclosure. On the basis of the above literature review, the main objective of the present work is to numerically study the effect of convective heat transfer for longitudinal radially divergent solid fins attached on the outer surface of the inner horizontal cylinder. The laminar natural convection heat transfer in the horizontal annulus is assumed to be under the steady state for an incompressible fluid (water). A total of six equally spaced and equal height fins with round tips which are made of highly conductive metal are considered. The inner cylinder is hotter than the outer cylinder so that the buoyancy induced natural convection flow occurs inside the annulus. It is expected that the existence of radial divergent fins will promote higher natural circulation inside the annulus compared to the other shapes of the fins. This geometry has potential applications in heat exchangers. The circumferential average Nusselt number can be directly used for designing the finned heat exchangers. The local Nusselt number, which is difficult to obtain accurately through experiments, is presented to gain a clear insight into the heat transfer processes locally. The main goal of this study is two folds, (1) compare the heat transfer results of the finned annulus with the bare annulus, and (2) develop suitable correlations with regard to Rayleigh number which can be used to predict the heat transfer rate within the investigated range of parameters.

2.0 MATHEMATICAL AND PHYSICAL MODEL

2.1 Physical Model of the Annulus with Solid Fins

The physical model of the investigated problem is illustrated schematically in Fig. 1, which consists of an annulus formed by two concentric horizontal cylinders. A total of six radially divergent solid fins of height H with round tip are equally spaced around the inner cylinder. The radius of the inner cylinder is denoted by r_i while that of the outer cylinder is denoted by r_o. The angle between two fins is 60°. The inner and outer cylinders are maintained at temperatures T_i and T_0 ($T_0 < T_i$), respectively. The cylinders are assumed to be long enough that a two dimensional analysis can be applied. The working fluid is water. Due to symmetry, the computations are carried for only half of the physical domain by making use of the vertical symmetry plane passing through the center of the cylinders. The number of fins showed in the computational domain is three. No fins are placed at the vertical symmetry plane.

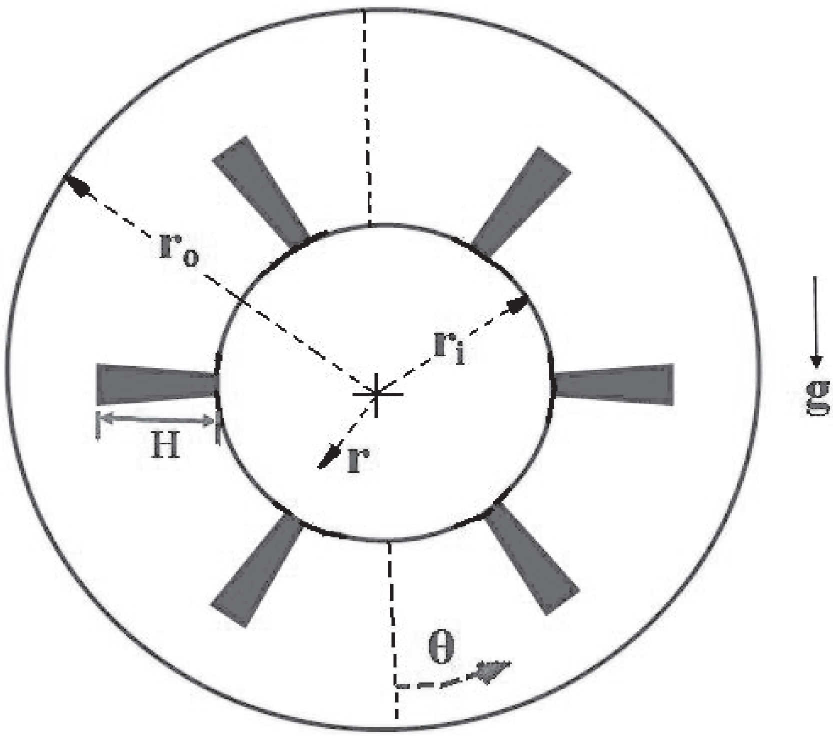


Fig. 1: Physical system and the cylindrical coordinates used to model the annulus with solid fins.

2.2 Mathematical Formulation

Assuming a steady state and two-dimension flow of a Newtonian incompressible fluid with the Boussinesq approximation for the thermal buoyancy term with negligible viscosity dissipation and radiation, the following non-dimensional form of the mass, momentum, and energy equations can be derived.
Upon invoking the Boussinesq approximation, the density can be written as follows:

$$\rho = \rho_{ref}\left[1 - \beta\left(\mathrm{T} - \mathrm{T}_{ref}\right)\right] \tag{1}$$

Continuity:

$$\frac{1}{R}\frac{\partial(RV)}{\partial R} + \frac{1}{R}\frac{\partial U}{\partial \theta} = 0 \tag{2}$$

U-momentum equation:

$$\frac{1}{R}\frac{\partial}{\partial R}(RUU) + \frac{1}{R}\frac{\partial}{\partial \theta}(UV) = \Pr\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial U}{\partial R}\right) + \Pr\frac{1}{R}\frac{\partial}{\partial \theta}\left(\frac{1}{R}\frac{\partial U}{\partial \theta}\right) - \frac{1}{R}\frac{\partial P^*}{\partial \theta} + 2\frac{\Pr}{R^2}\frac{\partial V}{\partial \theta} - \Pr\frac{U}{R^2}$$
$$-\frac{UV}{R} + Ra\Pr\phi\sin\theta \tag{3}$$

V-momentum equation:

$$\frac{1}{R}\frac{\partial}{\partial R}(RUV) + \frac{1}{R}\frac{\partial}{\partial \theta}(VV) = \Pr\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial V}{\partial R}\right) + \Pr\frac{1}{R}\frac{\partial}{\partial \theta}\left(\frac{1}{R}\frac{\partial V}{\partial \theta}\right) - \frac{\partial P^*}{\partial R} - 2\frac{\Pr}{R^2}\frac{\partial U}{\partial \theta} - \Pr\frac{V}{R^2}$$
$$-\frac{U^2}{R} - Ra\Pr\phi\cos\theta \tag{4}$$

Energy equation:

$$\frac{1}{R^2}\frac{\partial}{\partial R}(RV\phi)+\frac{1}{R}\frac{\partial}{\partial \theta}(U\phi)=\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \phi}{\partial R}\right)+\frac{1}{R}\frac{\partial}{\partial \theta}\left(\frac{1}{R}\frac{\partial \phi}{\partial \theta}\right) \tag{5}$$

On the solid fin wall the energy equation is:

$$\frac{K_S}{K_F}\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \phi}{\partial R}\right)+\frac{1}{R}\frac{\partial}{\partial \theta}\left(\frac{1}{R}\frac{\partial \phi}{\partial \theta}\right)=0 \tag{6}$$

The equations in the fluid domain are cast in dimensionless conservative form by introducing the following dimensionless variables:

$$R=\frac{r}{r_i}\ ;\ U=\frac{u}{\alpha/r_i}\ ;\ V=\frac{v}{\alpha/r_i}\ ;\ \phi=\frac{\mathrm{T}-\mathrm{T}_{ref}}{\mathrm{T}_i-\mathrm{T}_0}\ ;\ Ra=\frac{r_i^3\beta g(\mathrm{T}_i-\mathrm{T}_0)}{\nu\alpha}$$

$$p^*=\frac{\rho(r_i\,\mathrm{Pr})^3}{\mu^2}\left[P+\left(\rho_{\mathrm{ref}}(1-\beta\mathrm{T}_{\mathrm{ref}})-\frac{\mathrm{Pr}^2\rho^3\beta\mathrm{T}_{ref}}{\mu}\right)g\,r\cos\theta\right] \tag{7}$$

The detailed derivation of the above general transport equation can be found in ref. [10].

2.3 Boundary Conditions and Assumptions

The geometry (the right half of the annuli is chosen as the solution domain) and the dimensionless form of the boundary conditions corresponding to this problem are presented in Fig. 2.

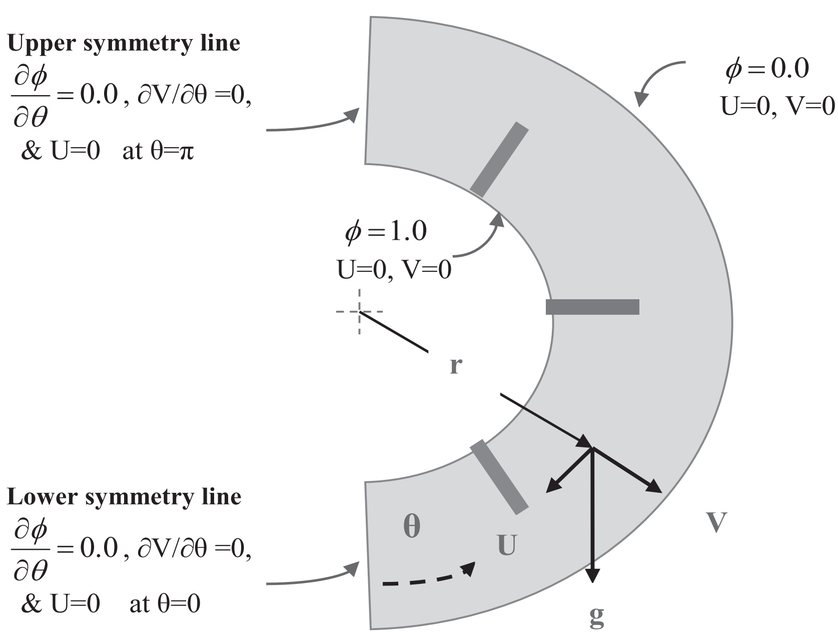


Fig. :. Schematic illustrations of the configuration with the boundary conditions.

On the solid fin surface: U = V = 0 ; $\frac{K_S}{K_F}=647.0$ (8)

Since all of the assumptions used here are provided in ref. [11], hence to avoid duplications those assumptions are not listed here. The details about the numerical solution procedure, the verifications of the code and the grid independent test all are available in [10, 11]. The convergence criterion is same as reported in ref. [11].

3. 0 RESULTS AND DISCUSSIONS

3.1 Governing Parameters

Figure 1 shows the geometry considered in this study. From the governing equations (2-6) and associated boundary conditions, one can see that the parameters which govern the problem are the number of fins, the orientation of fins, the fin conductance, the ratio of fin height to the annulus gap, Rayleigh number (Ra), Prandtl number, and the ratio of the outer radius to the inner radius. The first two governing parameters have been selected from the open literature, which were found by others [5] to be the best option so far the natural convective heat transfer is concerned for the inner cylinders. The dimensionless conductance of fin to fluid, and Prandtl number are assigned a value of 647.0, and 6.78, respectively. The working fluid is water. The ratio of the outer radius to the inner radius is assigned a value of 2.6 is taken from the literature. The remaining two parameters are varied over a wide range of values. One is the Rayleigh number which is assigned a value of 10^3, 10^4, 10^5, and 10^6. Other parameter is the ratio of fin height to the annulus gap which is assigned a value of 0, 0.2, 0.4 and 0.6.

3.2 Characteristic Parameters

The equations for the calculation of the local and circumferential average Nusselt numbers based on the inner cylinder radius, the circumferential average equivalent thermal conductivity ($\bar{K}_{eq}$) and local equivalent thermal conductivity K_{eq} along the surface of the inner cylinder, and the Nusselt number for conduction are given in [10, 11].

The local Nusselt number at the fin, based on radius, is given by:

$$Nu_f\left(\theta_f\right)=\int_1^{\frac{H}{r_i}}-\frac{1}{R}\left[\left.\frac{\partial\phi\left(R,\theta_f\right)}{\partial\theta}\right|_{TOP}+\left.\frac{\partial\phi\left(R,\theta_f\right)}{\partial\theta}\right|_{BOTTOM}\right]dR \tag{9}$$

The effective average Nusselt number, including the effect of the fin(s) is given by:

$$\overline{Nu_{Avg}}=\int_0^{\pi}Nu_{L\,\mathrm{at\,R=R_i}}(\theta)d\theta+\sum_1^{N=3}Nu_f\left(\theta_f\right) \tag{10}$$

where, N represents the number of fins. The average Nusselt number on the inner cylinder surface is obtained by integrating the local Nusselt number over the inner cylinder, which was obtained by using the Simpson integration rule. The average Nusselt number thus consists of the average Nusselt number on the inner cylinder and the effective average Nusselt number on the three fin surfaces.

3.3 Velocity and Temperature Fields

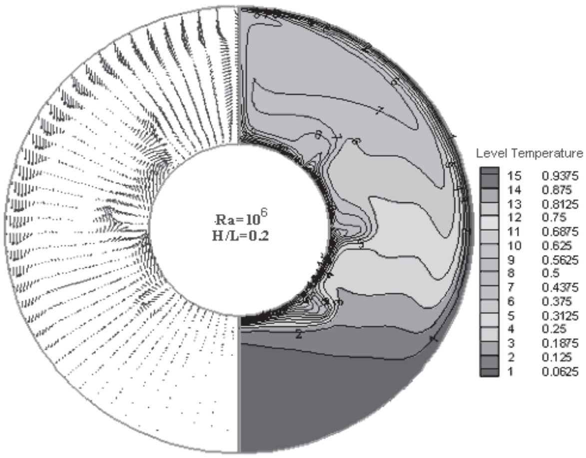
From the right half of Fig. 1 (computational domain), it can be seen that the first fin is located at an angular position of 30° with respect to the vertical symmetry plane while the second and third fins are separated by 60° from each other. Because of the above arrangements of the three fins, a total of four distinct compartments have been virtually formed. Since the orientations of the fins are different, as a result, the natural convection effect within a compartment differs from each other. Below the effects of the fins on velocity and temperature fields are discussed in the light of the above observations. In the present study, three ratios of fin height to the annulus gap are selected. To facilitate the presentation of the results, these three cases will be referred to as the short-fin, the intermediate-fin and the long-fin cases, respectively.

3.3.1. Effects of fin height

The results obtained for longitudinal divergent fin(s) inside the annulus are presented here. As mentioned before, fin(s) height is taken as 20%, 40%, and 60% of the difference in radii for the selected geometry. Flow and temperature fields in the form of velocity vectors (non-dimensional) and isotherms (dimensionless) are shown in Figs. 3(a-c) for the above three different fin heights at Ra = 10^6. There are a lot of differences found in the velocity vector patterns and isotherm distributions for three different fin heights. Results represent remarkable differences in contours especially in the upper part of the annulus and near the fins top and bottom surfaces and fins tips.

Fluid starts to move upward into the domain with an angular velocity around the inner cylinder, is shown in the left-hand side of Figs. 3(a-c). The fluid (water) approaches the bottom surface of the first fin gaining heat from the inner cylinder. The heated fluid then flows along the lower surface of the fin and gains more heat until it reaches the tip of the fin. Because of the orientation of the first fin with flow direction, this upward moving fluid impinges on the fluid moving along the lower surface of the fin and produces a stagnant zone near the bottom part of the root of the first fin. In the case of the short-finned annulus, the fluid that flows underneath the first fin is deflected into two streams near the tip of the first fin. One stream goes towards the colder surface of the outer cylinder while the other one moves toward the upper surface of the first fin. The latter one turns and then meets the heated fluid of the upper surface of the fin and moves upward by gaining heat from the surface of the cylinder and thus strengthens the convection flow in the gap between the first and the second fins (compartment-2). The above similar pattern is found for the cases of intermediate- and the long- finned annuli. At this point the flow streams are deflected, a portion of the flow streams move into the third compartment, while the remaining portion joins the cold fluid at the outer cylinder. The bottom surface of the second fin (horizontal) produces a thermally stratified situation because of a stable density gradient there. As a result, the low conductive mode of heat transfer prevails underneath the second fin surface. Whereas, the flow in the third compartment guided by the top surface of the second fin, generates an unstable density gradient and thereby causing a stronger convection due to the development of the Benard-type of convection. The top surface of the horizontally positioned fin thus significantly enhances the heat transfer rate from the base to the tip of the fin. Then the fluid rises along the inner cylinder until it reaches to the third fin. The third fin, since, it is upwardly inclined with respect to the gravity field it produces a favorable natural convection underneath which is aided by the heated fluid coming from the third compartment. In the short-, the intermediate- and the long-finned annuli, the combined flow attains a high momentum which is directed radially towards the colder outer surface. The cold fluid that flows downward along the inner surface of the outer cylinder from the top is thus subjected to the resistance which practically depends on the height of the fin; the longer the fin the higher the resistance force meet by the cold fluid. This cold fluid is thus directed radially inward and is forced to turn and recirculate in the fourth compartment. For the short-fin annulus, the buoyant fluid that comes from the third compartment does not have enough momentum to reverse the downward flow of the cold fluid completely. However, for all the three fin heights the flow of the cold fluid is not strong enough to overcome the convection that develops at the root of the fin and as a result, a separation of fluid occurs there. It may be noticed that near the upper symmetry plane, the flow changes its direction. Most of the flow near the inner cylinder follows anticlockwise angular direction and rest of the angular component is converted to a tangential one and it moves in the downward direction along the outer cylinder. In the intermediate- and long-fin geometries, most of the fluid flow is confined in the gap between the fins. At a higher fin height, the fluid velocity adjacent to the fins is higher because of gaining more heat compared to the lower fin height. The lower height fins results in a smaller stagnant zone which facilitates the fluid to move from gap to gap faster, and thus can't accelerate the heat transfer rate.

The right hand side of Figs. 3(a-c) shows the isotherm contours in flooded format inside the annulus for three different fin heights all at Ra = 10^6. The heat transfer between the two cylinders depends on the flow velocity, the surface area, and the flow patterns. So, higher velocity leads to a higher heat transfer, and larger surface area also enhances the heat transfer rate. As shown in these figures, the existence of the first solid fin resists the natural circulation in the first compartment of the annulus. This obstacle increases the thermal boundary layer along the bottom surface of the inner cylinder and the attached fin. A predominantly conduction mode of heat transfer prevails in the first compartment. The conductive heat transfer rate is enhanced as the fin height increases. The flow through the second, third and top compartments, as shown in these figures, has destroyed the thermal boundary layer along the upper part of the fins and has decreased the boundary layer thickness along the outer surface of the inner cylinder. These figures indicate that as one moves from the lower compartment to the adjacent upper compartment the heat transfer rate significantly enhances which can be observed from the locations of the corresponding isotherms.



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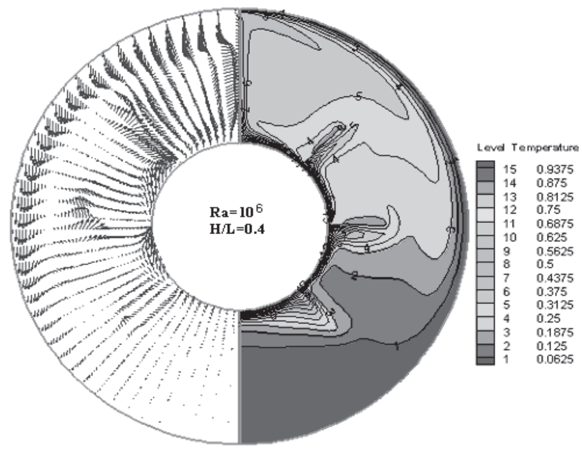

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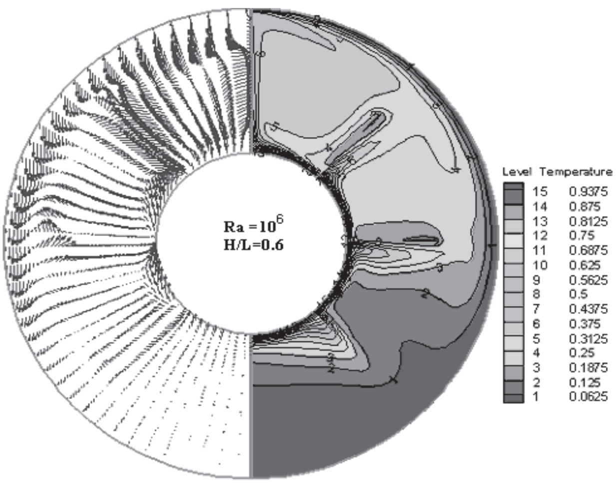


c

Fig. 3: Velocity vectors and isotherms (non-dimensional) for three different fin heights at Ra=10^6

In the annulus geometry, the hot upward flow at the top of the inner cylinder gets cold upon impingement on the cold outer wall. The local heat transfer rate is high at this region of impingement. The region of impingement enlarges in case of long-fin geometries. Besides this, the stagnant zone due to the fins is remarkably efficient for the long-finned geometry compared to other two cases. Due to the blockage, fluid can contact closely with the outer wall of the inner cylinder, thus fluid can take away more heat for long-finned annulus. The upward motion of the hot fluid causes the isotherms to be packed densely at the upper part of the outer and lower part of the inner cylinders. At the top of the inner cylinder, a typical type of plume is seen to have been developed for all the studied cases as expected. The strength of the plume increases with the increasing height of the fin.

3.3.2. Effects of Rayleigh number (Ra)

Figures 4(d-g) and Figs. 5(h-k) present the flow patterns and temperature distributions inside the annulus for H/L of 0.2 and 0.6 for Ra varying from 10^3 to 10^6. In the preceding paragraph, the flow pattern which is developed due to the presence of fins in an annulus has already been explained and there is no need to describe the phenomenon again. These figures represent that for a fixed fin height, by increasing the Rayleigh number the fluid motion becomes stronger and are consistent with the physics of the problem. When the Rayleigh number increases, the isotherms follow the contour of the outer cylinder more closely as well as the isotherms are compacted toward the outer wall of the inner cylinder. The usual manifestation of the isotherms prevails with the increase of the Rayleigh number indicating that the convective mode of heat transfer is increasing with the Rayleigh number. This indicates that the local heat transfer coefficient becomes non-uniform with the increase of Ra. For a fixed value of Ra, the isotherm contours depict that by increasing the fin height from 20% to 60% of the difference in radii the isotherms expand faster radially towards the outer cylinder, and leads to a significant enhancement of heat transfer rate therein, as expected. In the short-finned case, the fins are too short to block the flow between the two gaps of the fins. Besides this, only small impingements are observed at the tip and at the base of the fins. Again, the flow blockage due to the fins can be seen in the long-fin geometry, especially at high Rayleigh numbers. The results indicate that even at higher Ra and for greater fin height a considerable portion of the water layer still remains colder in the region below the first fin. The natural convection effect becomes weak there, which is in contrast with the phenomenon observed in ref. [7]. Neither the increase of Ra nor the increase of fin height eliminates the stable heat transfer induced by the heat conduction especially at the bottom section of the annulus.

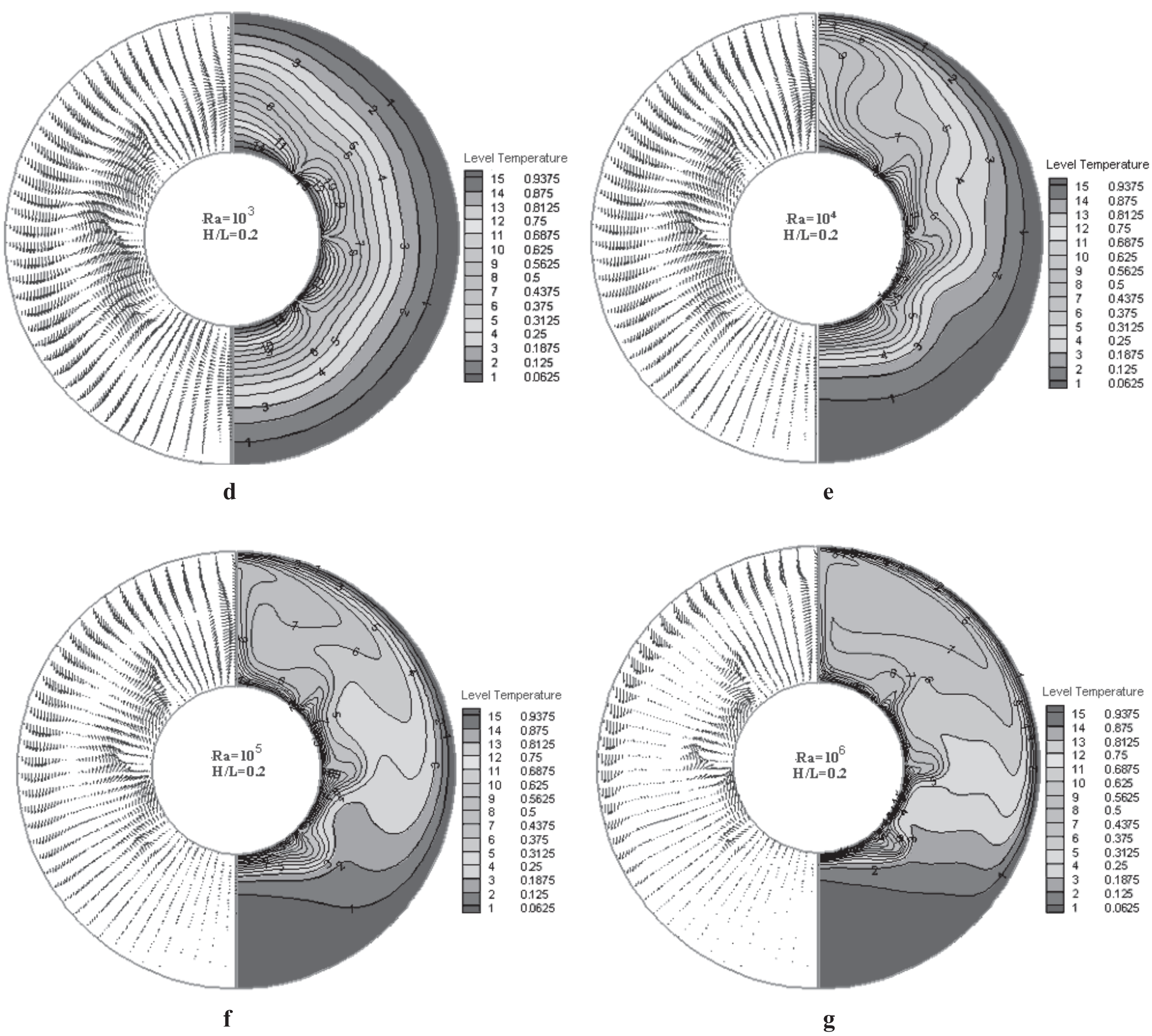


Fig. 4: Velocity vectors and isotherms (non-dimensional) for four different Rayleigh numbers for H/L=0.2

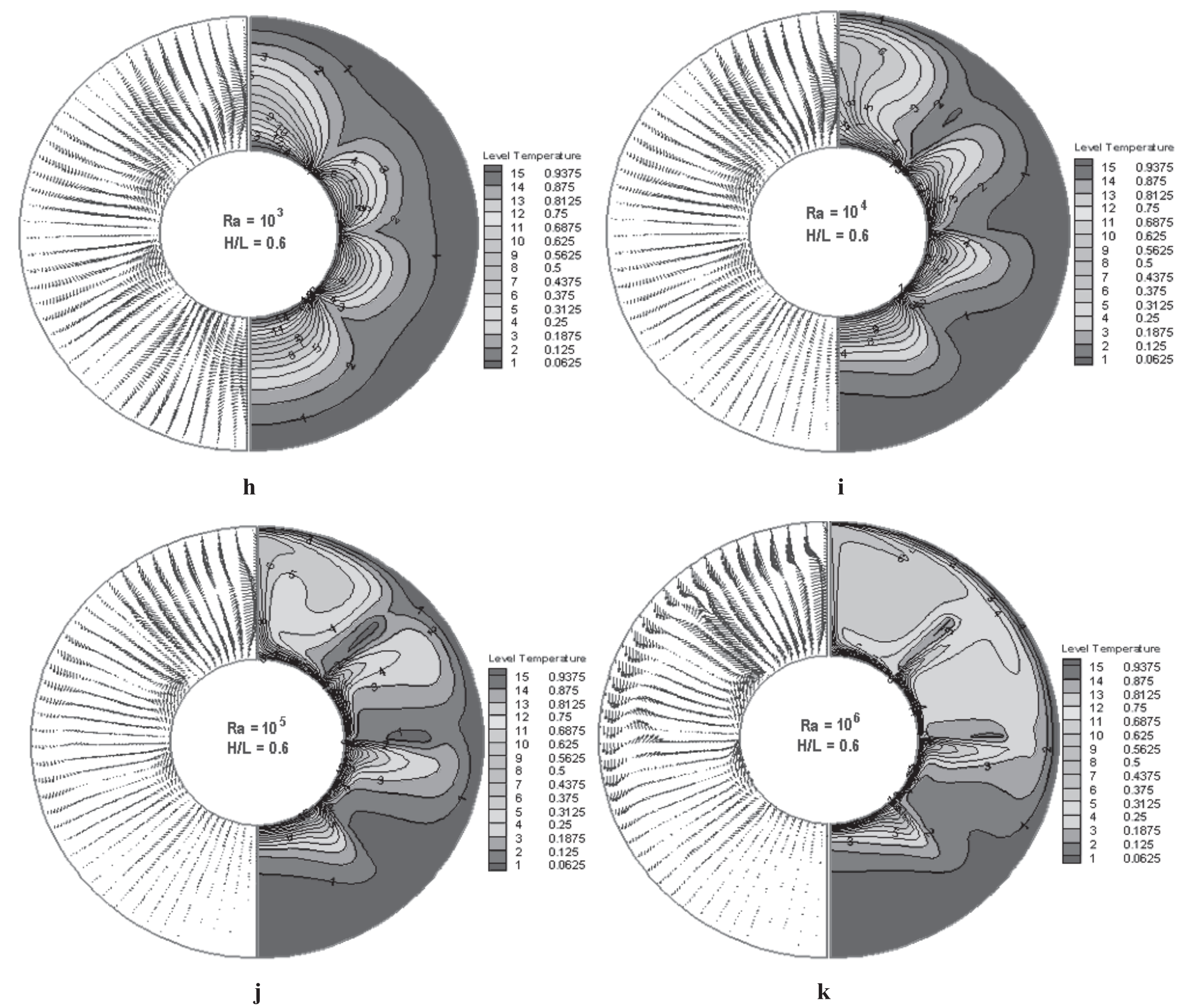
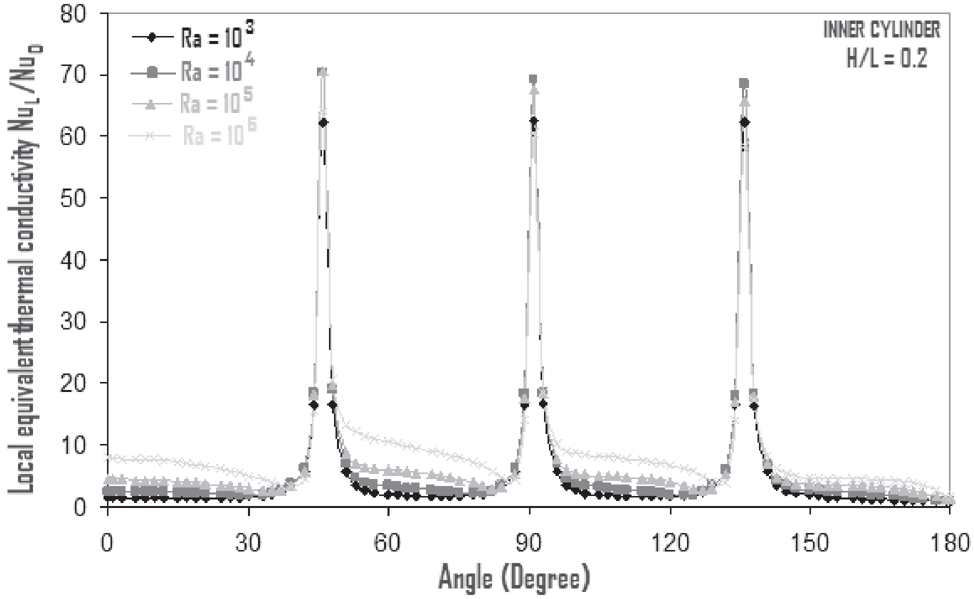


Fig. 5: Velocity vectors and isotherms (non-dimensional) for four different Rayleigh numbers for H/L=0.6.

3.4. Distribution of the Local Equivalent Thermal Conductivity

The local equivalent thermal conductivity is presented in Figs. 6(l-n) for the surface of the inner cylinder for H/L of 0.2, 0.4, and 0.6 when the non-dimensional radius ratio is 2.6. These figures provide the values of the local equivalent thermal conductivity of internally attached finned geometry on the inner cylinder for $10^3 \leq Ra \leq 10^6$. These figures show a trend which is similar to the trend of the figures for an internally finned horizontal annulus available in the literature [5]. For comparison purposes, the local heat transfer results for fin height to annulus width of 0.2, 0.4, and 0.6 and for Ra of 10^4 to 10^6 are given in Figs. 7(o-q). A careful inspection of the figures shows that for a given fin height, the local equivalent thermal conductivity increases with the increase of the Rayleigh number except in the case of fin height to annulus width of 0.2 for which it shows a non-monotonic behavior against Rayleigh numbers. For Ra = 10^4, a maximum value of local equivalent thermal conductivity is observed at the fin tips, which decreases with the increase of Ra. This behavior indicates that for a specific geometry, there exists an optimal fin height for the highest heat transfer rate when the number of fins is specified. It is observed that the flow circulation at Ra = 10^3 is very weak for all fin heights, leading to a temperature distributions similar to the conduction situation (see Figs. 4(d), and 5(h)). The pattern of local equivalent thermal conductivities with respect to angular movement for H/L of 0.2, 0.4, and 0.6 is similar. From the base of the first fin to the base of the third fin, the local equivalent thermal conductivity increases first and it reaches the highest value at the tip of the fin and then it decreases rapidly along the top surface to the base of the fin. Here, the local equivalent thermal conductivity starts from a higher value for the top of the fin-tip and decreases much faster to a low value at the base of the fin. It is quite remarkable to see the value of the local equivalent thermal conductivity remains practically unchanged from fin to fin.



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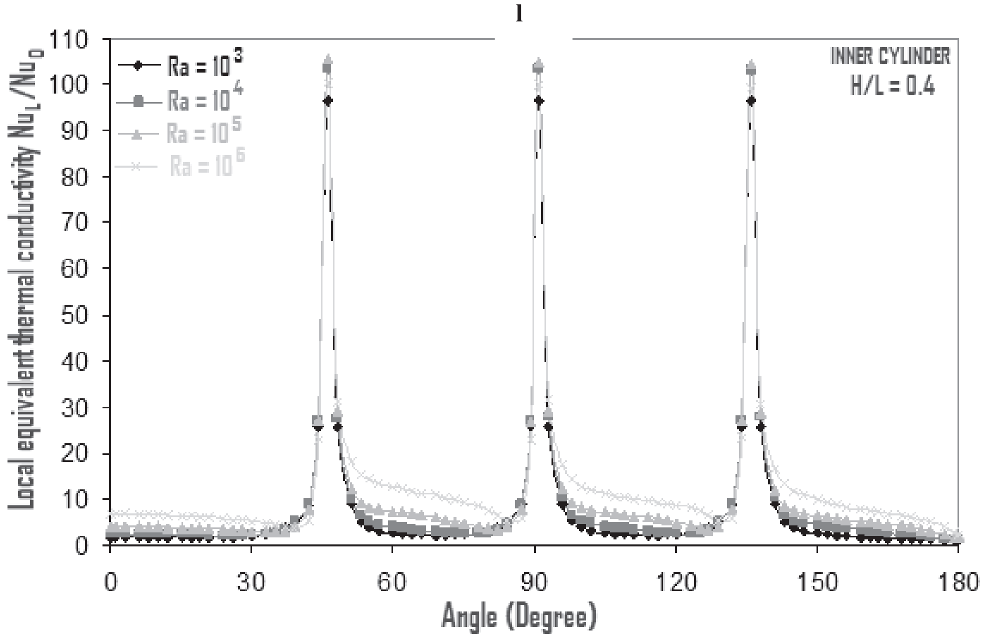

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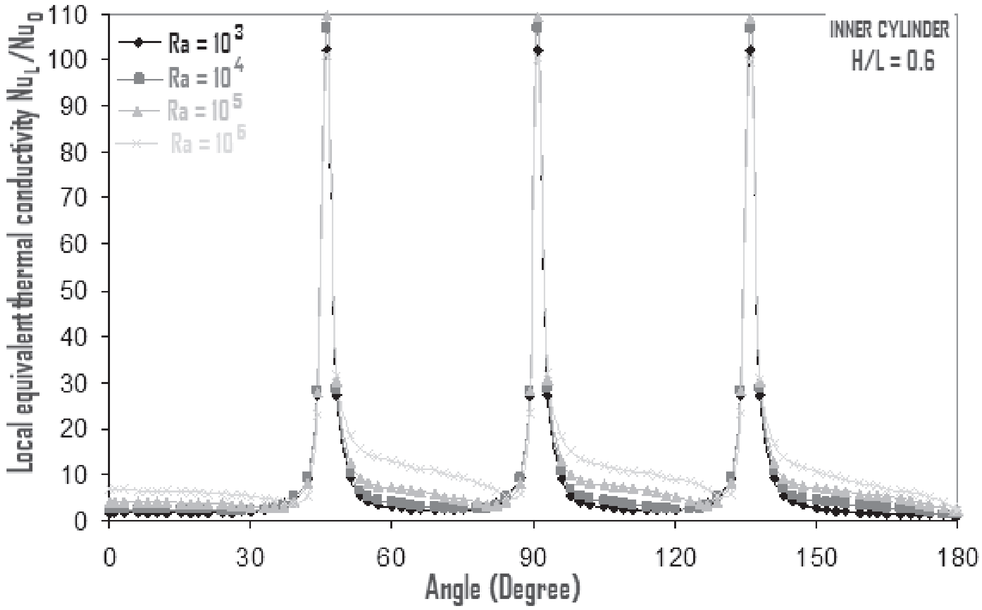

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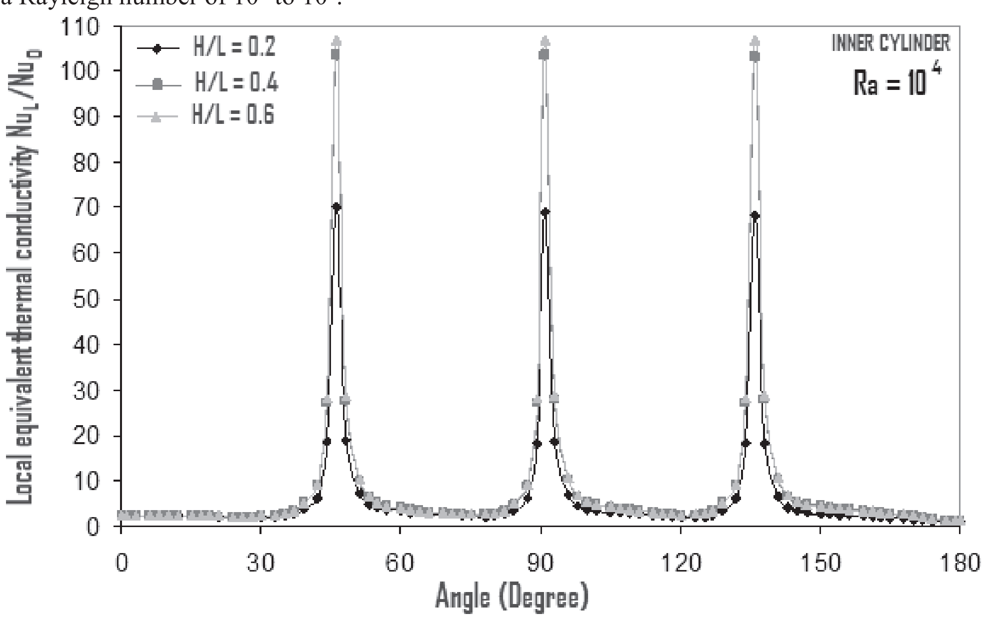
Fig. 6: Local Nusselt number variation for the case of fin height (H) at 20%, 40%, and 60% of radius difference (L) for a Rayleigh number of 10^3 to 10^6.



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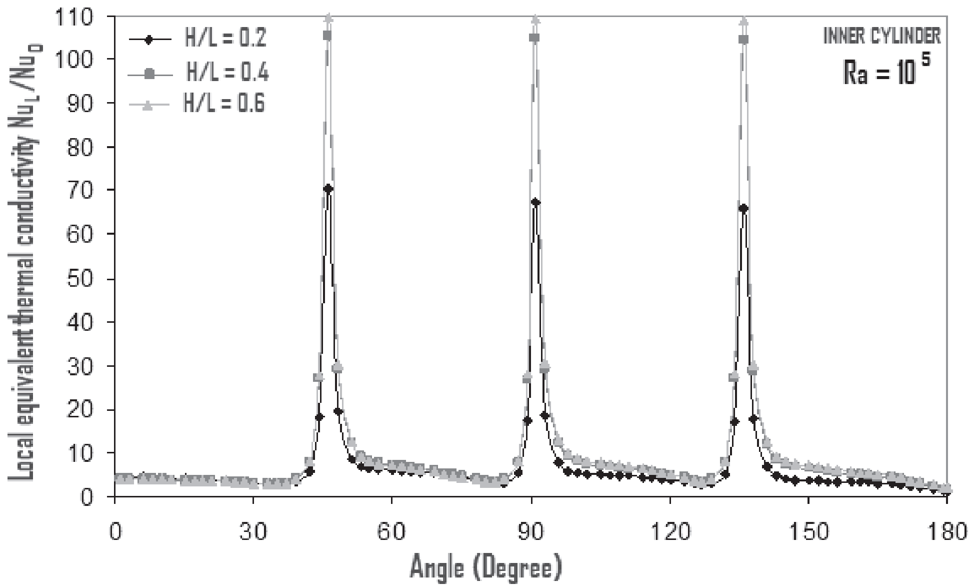

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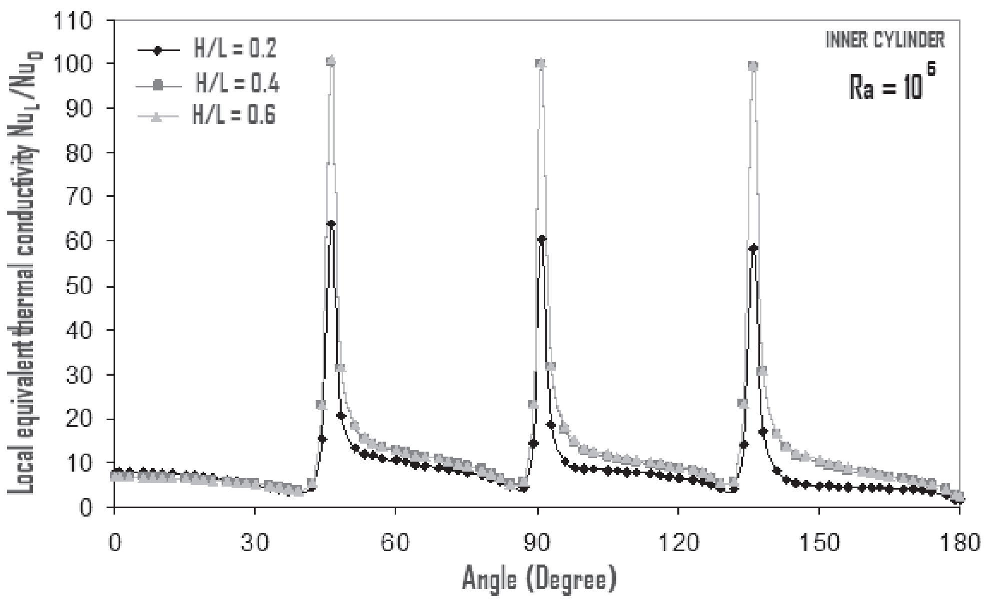

q

Fig. 7: Comparison of Local Nusselt number variation for the case of fin height (H) at 20%, 40%, and 60% of radius difference (L) for a Rayleigh number of 10^4 to 10^6

This behavior of the local equivalent thermal conductivity can be explained as follows: in regions near the base of the fins, the flow is virtually stagnant due to the resistance imposed on the flow by the fin wall. The fluid in this part attains or nearly attains the temperature of the inner cylinder and fin walls, reducing the transfer of heat. This leads to a near-zero local equivalent thermal conductivity. As fluid moves away from the base, it gains momentum, leading to continuous increase in the velocity. The increase in the velocity removes heat from the wall more effectively. As a result, the heat transfer rate increases as one move towards the tip. And another reason for increasing the heat transfer rate, as we move away from the base of the fin, is the impingement of water on the fin surface which is clearly noticeable at Ra = 10^6 in Figs. 3(a-c). The impingement of cold fluid on the hot fin at the fin-tip creates a region of high rate of heat transfer. For all fin heights, in this region a highest heat transfer rate is observed which reveals through the densely packed isotherms at the fin tips as seen in Figs. 4(d-g) and 5(h-k). Local equivalent thermal conductivity then decreases as it approaches the upper part of the inner cylinder. This is because the heated fluid gradually gets detached from the inner cylinder wall. This region is ordinarily known as the plume region. Figures 7(o), 7(p) and 7(q) show the comparison of the local equivalent thermal conductivity for different ratios of fin height to radius difference at Ra = 10^4, 10^5, and 10^6, respectively. The local equivalent thermal conductivity increases with the increase of the fin height no matter what the Rayleigh number is except at the fin-tip for H/L = 0.4 and 0.6 and Ra = 10^6.

3.5. Average Equivalent Thermal Conductivity

Figure 8 shows the average equivalent thermal conductivity for three values of H/L as a function of Ra and for the horizontal annulus with no fins (referred to as plain annulus). The average Nusselt number increases with both the increasing Ra and increasing the fin height. The plain annulus case is similar to the solid fin cases, i.e., with the increase of the Rayleigh number the heat transfer rate increases. Each graph shows three distinct regimes of heat transfer. The first regime is the conduction dominated region where the average equivalent thermal conductivity increases slowly with the increase of Ra. The second and the third regimes are the transition regime and convection dominated regime, respectively, where the average equivalent thermal conductivity is strongly dependent on the Rayleigh number. As discussed earlier, Figs. 3(a-c) provides the flow patterns and isotherms for different fin heights. We can easily see that the highest fin generates more convective flow within the fins gap which contributes more to the heat transfer. The highest fin due to the increased surface area transfer heat more efficiently compared to the fins with lower heights and it produces the highest heat transfer rate.

As shown in this figure, the heat transfer rate from the inner cylinder surface for the plain annulus is the lowest among the four studied cases. The works in refs. [1-3] which have dealt with a plain annulus show that a typical kidney-shaped flow circulation develops where the flow originates from the lower part of the inner cylinder and moves upward gaining heat from the inner cylinder and reaches the top. The heated fluid then turns down due to the restricted gap at the top and moves downward along the inner surface of the outer cold cylinder. Thus a recirculation zone develops within the annulus where the velocity is very low at the circulation centers. With the increase of the Ra the circulation center moves upward.

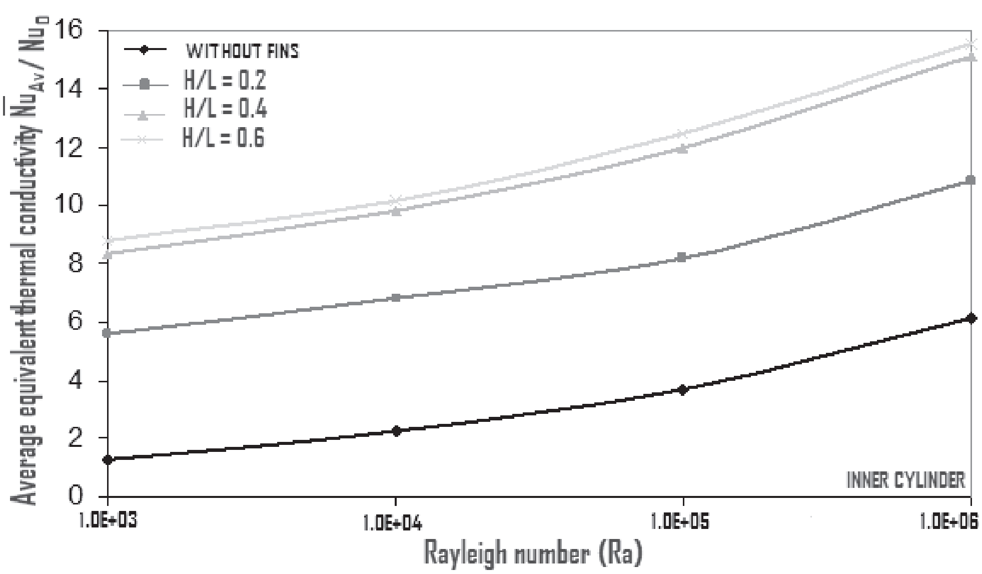

Fig. 8: Buoyancy effect on heat transfer from a plain and finned inner cylinder

In the plain annulus, the heat transfer is restricted along the bottom part of the inner cylinder and at the upper part of the annulus. As a consequence, the natural circulation of heat at the bottom part of the annulus decreases remarkably. On the contrary, the attachment of the solid fins on the inner cylinder intensifies the heat transfer along the outer cylinder, along with the bottom part of the inner cylinder and near the attached fins. The existence of the fins decreases the thickness of the thermal boundary layer along the bottom part of the inner cylinder surface. This decrease in the thermal boundary layer is more pronounced as the fin height increases and as a result, it promotes a greater heat transfer rate.

The heat transfer data for water are correlated using a least-squares regression analysis and are expressed below:

$$\overline{K}_{eq_{inner}} = 0.2695\ Ra^{0.2272}, \quad 10^3 \leq Ra \leq 10^6, \text{ for plain cylinder (no fins)}; \quad R^2 = 99.9\% \tag{11}$$

$$\overline{K}_{eq_{inner}} = 2.8602\ Ra^{0.0946}, \quad 10^3 \leq Ra \leq 10^6, \text{ for } H/L = 0.2; \quad R^2 = 99.1\% \tag{12}$$

$$\overline{K}_{eq_{inner}} = 4.5322\ Ra^{0.0858}, \quad 10^3 \leq Ra \leq 10^6, \text{ for } H/L = 0.4; \quad R^2 = 99.3\% \tag{13}$$

$$\overline{K}_{eq_{inner}} = 4.8448\ Ra^{0.0833}, \quad 10^3 \leq Ra \leq 10^6, \text{ for } H/L = 0.6; \quad R^2 = 99.1\% \tag{14}$$

From the above correlations, it is evident that, as the fin height increases, the Rayleigh number exponent decreases and the value of the constant multiplying factor increases. The decrease in the exponent illustrates that the fin makes the flow less global. This means that when the fin height increases the communication in the cavity decreases and leads to the formation of small convection cells by partitioning the flow. The adequacy of the regression model listed is above 99% for all three fin heights and for plain annulus. For $Ra = 10^6$, three fins of dimensionless height to annulus width ratio 0.6 increase the average equivalent thermal conductivity by 154% over its value for the plain annulus, whereas the corresponding increase for two fins of dimensionless height to annulus width ratio of 0.4 and 0.2 are 147%, and 77%, respectively, indicating it is better to use higher fin height.

4.0 CONCLUDING REMARKS

The problem of steady laminar natural convection in a horizontal concentric finned annulus is studied numerically. The outer surface of the inner cylinder is fitted with six internal longitudinal divergent solid round tip metallic fins which are positioned at an equal angular spacing of 60°. The flow and heat transfer between the concentric horizontal cylinders have been studied for Ra up to 10^6 and for the fin length of up to 0.6 times the annulus gap of the cylinders considering water as the working fluid. The following major conclusions can be drawn from this study:

[1]. For all the three fin heights, the results indicate that heat transfer rate increases with increasing Rayleigh number due to enhanced natural convection inside the annulus.

[2]. The heat transfer rate from the inner cylinder surface is much higher for the solid-finned annulus compared to the plain annulus because of the increased temperature gradient and favorable thermal buoyancy effect there.

[3]. For a fixed Rayleigh number, the heat transfer rate increases with the increase in fin height. The larger the fin height the higher is the stagnant zone and there is less communication of fluids in the cavity due to the increased partitioning of the flow into small convection cells bounded by the fins. The highest fin has more surface area compared to the shorter fins. All of the above factors lead to the enhancement of the heat transfer rate between the two concentric horizontal cylinders under natural convection.

[4]. The effect of fins on the heat transfer rate is lower at the bottom part of the annulus below the first fin i.e., in conduction-dominated zone compared to the upper part of the annulus i.e., in the convection-dominated zone. The above statement is especially true at higher Rayleigh numbers for the studied solid-finned geometry. This is because for higher Rayleigh numbers the heat transfer is concentrated near the inner cylinder and as a result, the fluid coming from the top and moving downward along the outer cold cylinder can not gain heat from the fluid near the inner cylinder.

In Part-II of the continuing study, the effect of the porous fins on the flow and heat transfer in the cylindrical annulus have been studied and a comparison is made with regard to the heat transfer rate between the solid fins studied in Part-I and porous fins studied in Part-II.

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